

# Courage to Risk – 1.20.2023



## Understanding Students' Mathematical Development: Let's Do This!

Mindy Adair, Ph.D.

Director of Mathematics – Denver Academy

[madair@denveracademy.org](mailto:madair@denveracademy.org)

[@AdairMindy](https://twitter.com/AdairMindy)

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## About me...



Self Proclaimed: Selfie Expert!



MINDY ADAIR, PH.D.

THOUGHTS:

WHICH ONE?

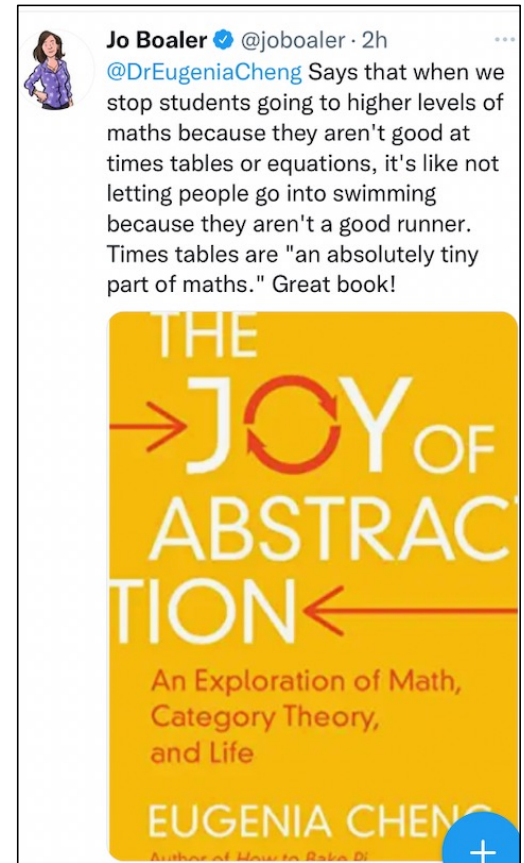
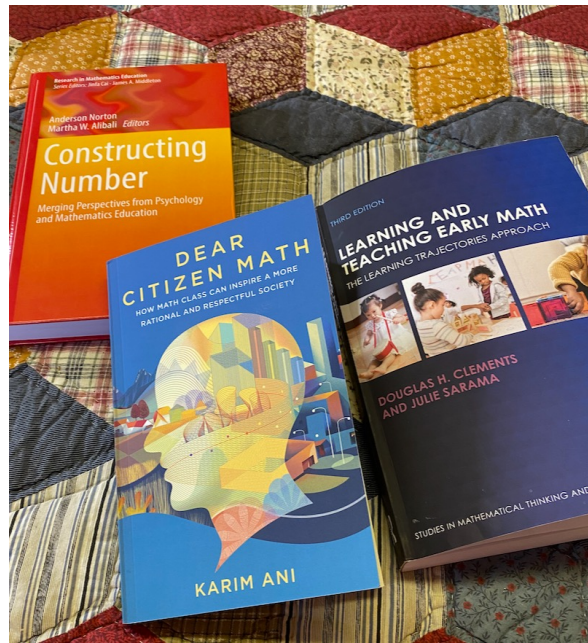
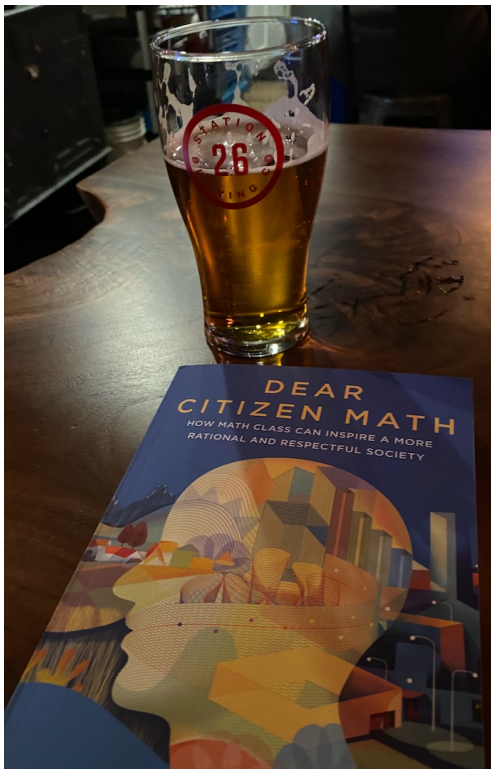


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# Research...




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## More Research



“Children with or at risk for disabilities start to enjoy math when they start to see small successes with their learning. Math can be for all!”



**GRANTEE SPOTLIGHT**  
**Dr. Sarah Powell**  
*Associate Professor  
The University of Texas at Austin*

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A vertical decorative bar on the left side of the slide, composed of three stacked rectangular segments. The middle segment is a lighter blue and contains the Greek letter pi ( $\pi$ ). The top and bottom segments are a darker blue.  $\pi$ 

## Three goals for us today...

1. We will do Maths.
2. We will consider Mathematical Development.
3. You will leave here knowing how funny I am.

(Cause I am – funny. Just sayin’)



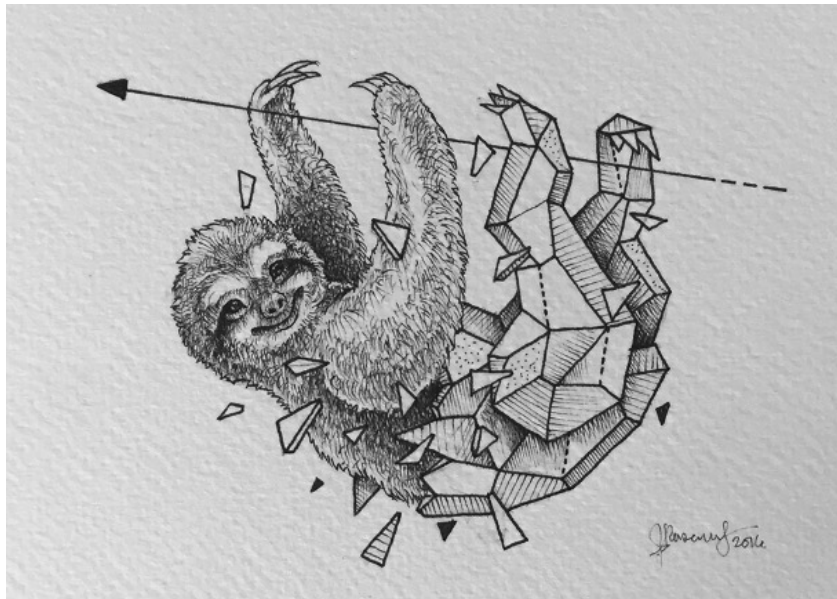
## Understanding Students' Mathematical Development: Let's Do This!

### Agenda:

- › Introduction: Mathematical Development – How does this really work?
- › Warm Up.
- › Numbers are complicated! How do students construct meaning with numbers?
- › Support for Students: Let's learn some maths!
- › Five Tasks to Do.
- › Three Activities for Class on Monday.
- › Closing.

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Welcome & Thank you for coming!



Kerby Rosanes

<https://kerbyrosanes.com/geometric-beasts>

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Warm Up:

The answer is...2023.

A. What is one question?



**Howie Hua**  
@howie\_hua  
[howiehua.wordpress.com](http://howiehua.wordpress.com)

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Warm Up:

The answer is...2023.

B. What is the prime factorization of 2023?



$\pi$

Warm Up:

The answer is...2023.

B. What is the prime factorization of 2023?



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Warm Up:

The answer is...2023.

C. How can you use the set of numbers from 1 to 10 and mathematical operations to get 2023?



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Warm Up:

The answer is...2023.

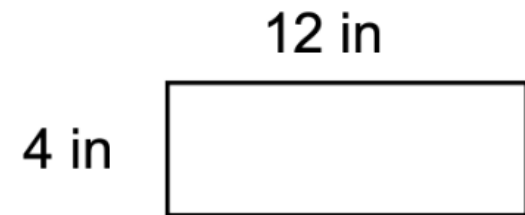
C. How can you use the set of numbers from 1 to 10 and mathematical operations to get 2023?

Solution to this challenge at the end of the session!

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The questions we ask – matter!

Example (Jo Boaler):



1. Find the area of the rectangle:

Versus

2. Find all rectangles with area 48 inches squared.

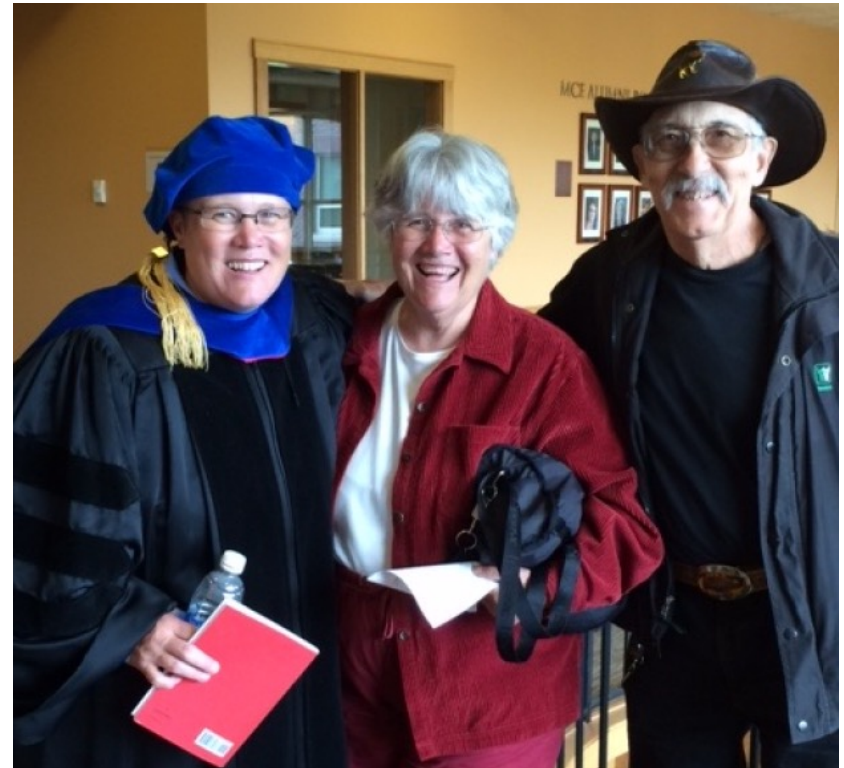
How does question #2 differ from question #1?

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I learned to ask better questions...

My parents...

Bob and Gale!



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Sometimes we really need to know, before we go!





# Learning versus Performance Opportunities

Perhaps we consider:

1. Give a student a problem with the answer.

Ask: “What is the process/thinking needed to get that answer?”

2. Rather than asking: “Does anyone have a question?”

Ask: “Think of one question that someone in the room might ask.”

# Performance versus Learning Opportunities

## PERFORMANCE

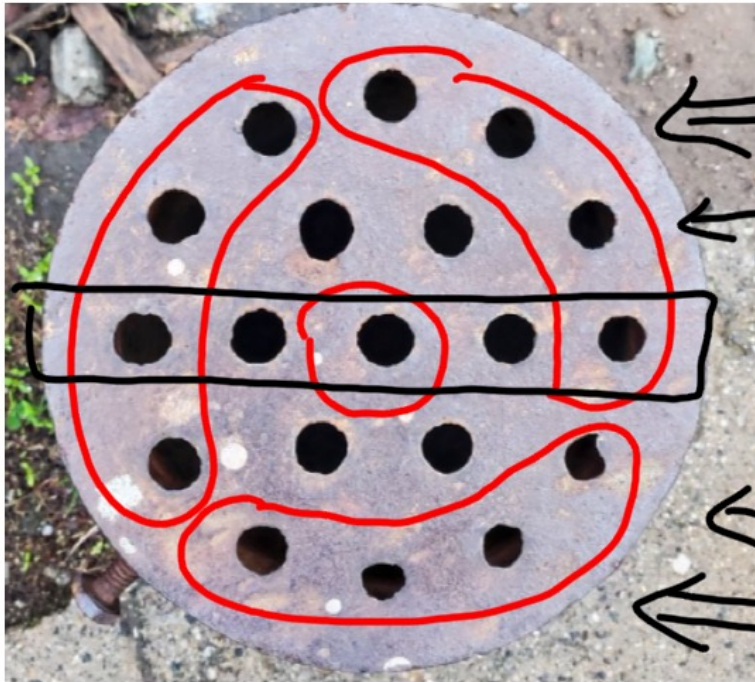
- Math Fact Mad Minutes
- Quizzes & Tests
- Mass Practice
- Standardized Tests

## LEARNING

- Tasks
- Number Talks
- Conversations
- Interleaving Instruction
  - Fractions, Decimals, and Percentages (Not Blocking)

Bjork, Elizabeth & Bjork, Robert. (2011). Making things hard on yourself, but in a good way: Creating desirable difficulties to enhance learning. *Psychology and the Real World: Essays Illustrating Fundamental Contributions to Society*. 56-64.

## Warm Up: How Many? How did you count?



We can access and insight into student thinking with these kinds of learning opportunities!

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## Task #1:

What do you notice? What do you wonder?





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## Reflection – in your journal:

1. What is maths?
2. Why do we need maths?
3. How is maths important to:
  - a) Students?
  - b) Schools?
  - c) Societies?

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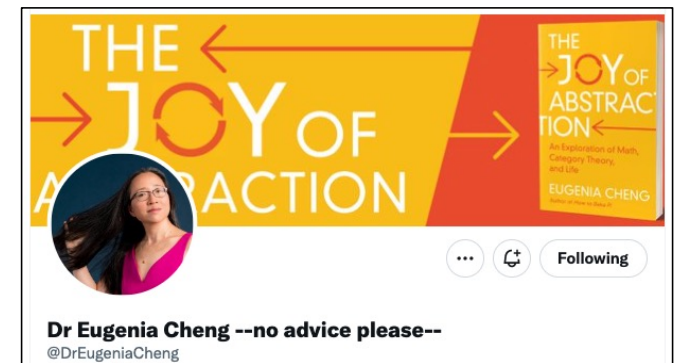
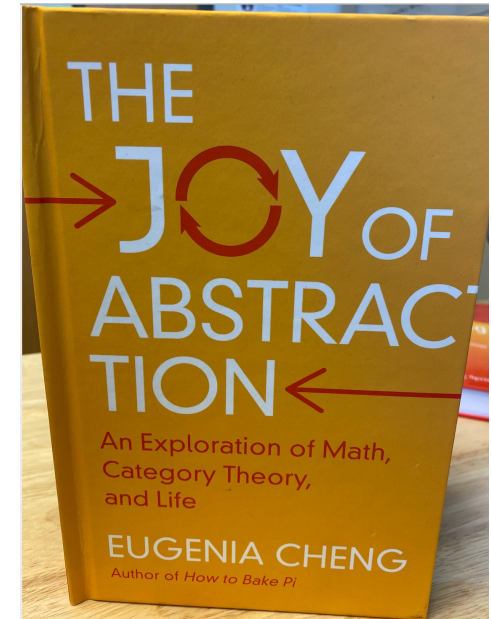
# Mathematics – what is it?

@DrEugeniaCheng

“Math has an image problem.

Many people are put off by it in school and end up as adults either hating it, being afraid of it, or defensively boasting about how bad they are at it or how irrelevant it is anyway.”

Cheng, E. (2022). *The Joy of Abstraction: An Exploration of Math, Category Theory, and Life*. Cambridge: Cambridge University Press. P. 2



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## The Status of Mathematics:

“I think there are broadly three reasons math education is important:

1. As a foundation for further study in mathematical fields.
2. For direct usefulness in life.
3. To develop a particular way of thinking.”

Cheng, E. (2022). *The Joy of Abstraction: An Exploration of Math, Category Theory, and Life*. Cambridge: Cambridge University Press. P. 2

## The Status of Mathematics:

“I think there are broadly three reasons math education is important:

1. As a foundation for further study in mathematical fields.

This is not relevant for many people.

2. For direct usefulness in life.

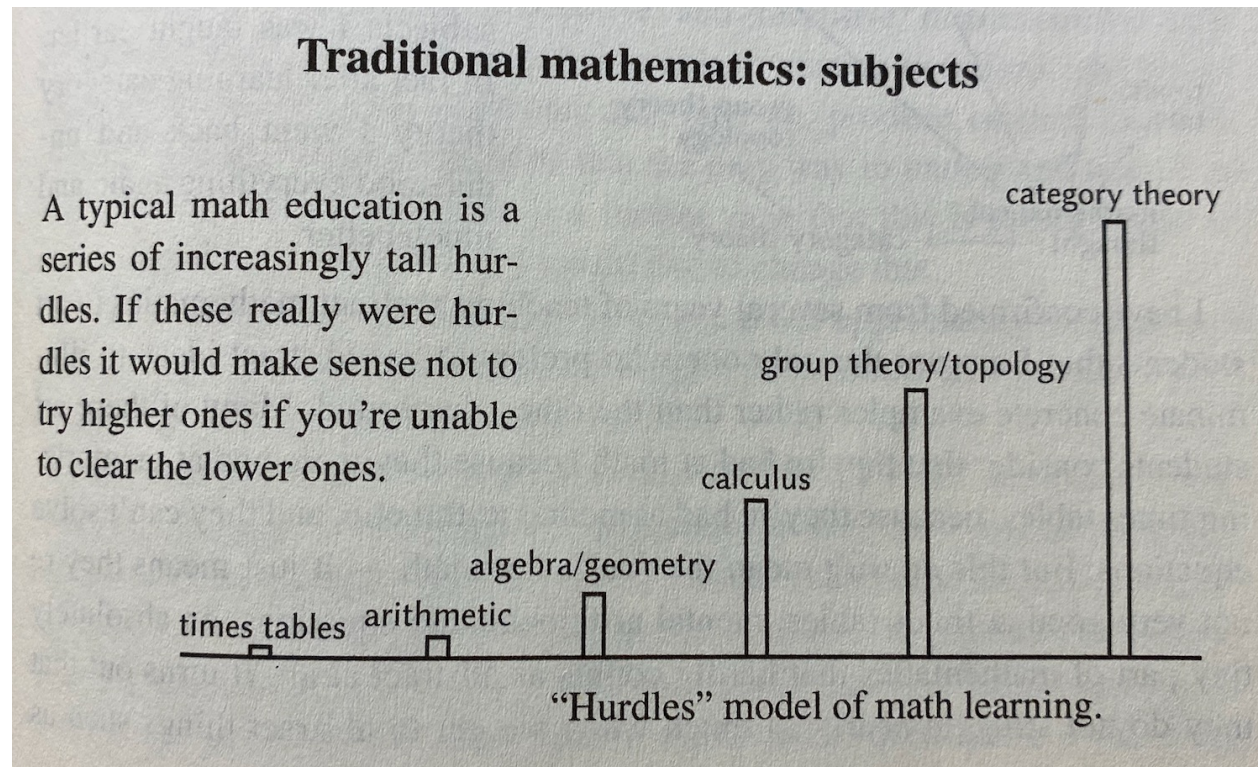
Focusing on arithmetic without calculators is not ideal. Having some quantitative literacy is important – mortgages, interest rates, taxes.

3. To develop a particular way of thinking.

Making connections between diverse situations is empowering.

Cheng, E. (2022). *The Joy of Abstraction: An Exploration of Math, Category Theory, and Life*. Cambridge: Cambridge University Press. P. 2

# Mathematics – what is it?



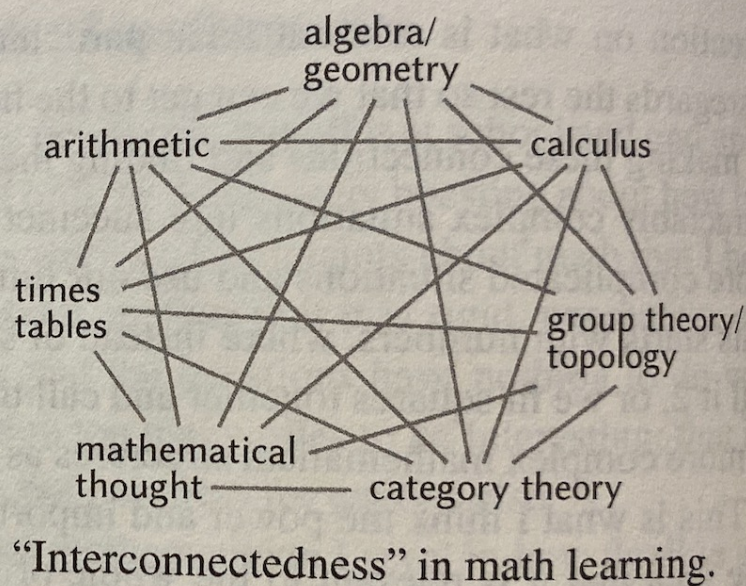


# Mathematics – what is it?

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## Prologue

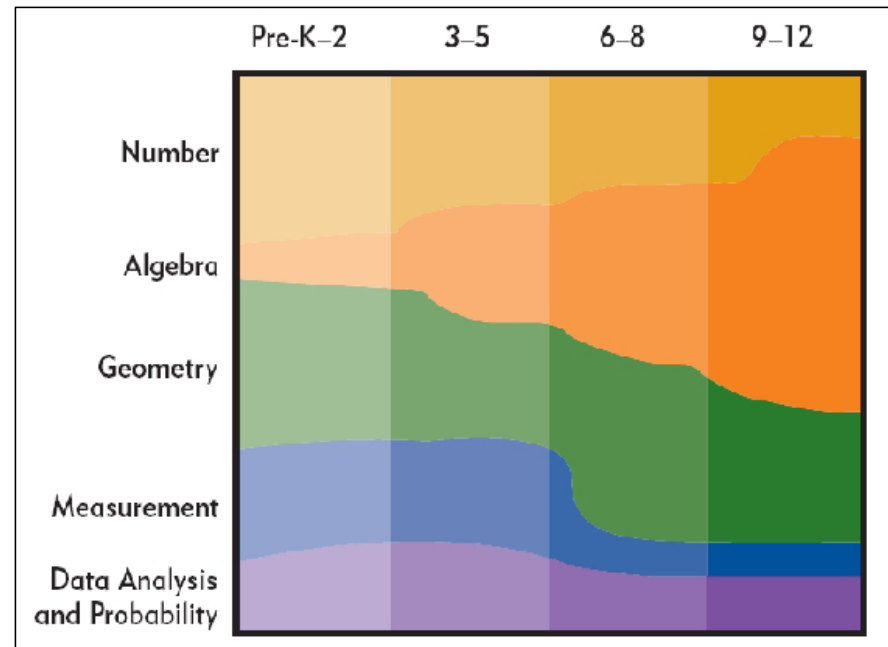
However, math is really more like an interconnected web of ideas, perhaps like this; everything is connected to everything else, and thus there are many possible routes around this web depending on what sort of brain you have.





# NCTM Content Standards

1. Numbers and Operations
2. Algebra
3. Geometry
4. Measurement
5. Data Analysis and Probability



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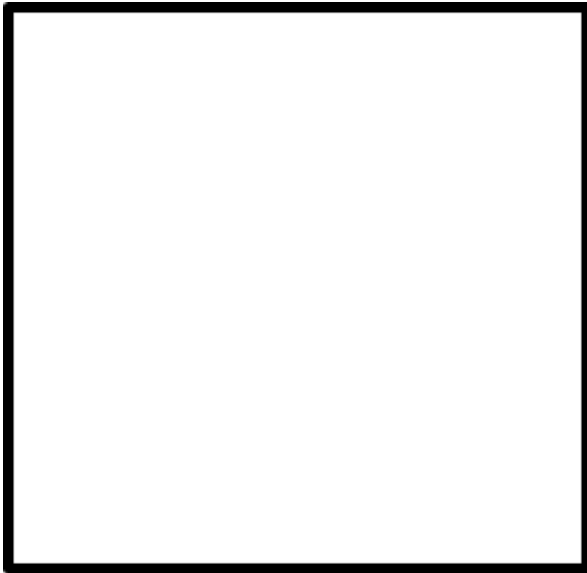
## Task #2:

Task: Divide the square into four equal parts...as many ways as you can.



## Task #2:

Task: Divide the square into four equal parts...as many ways as you can.



What is interesting about this task?

How is this task accessible to all students?

LFHC = Low Floor High Ceiling Tasks...

Possible Solutions:

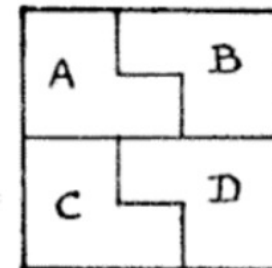
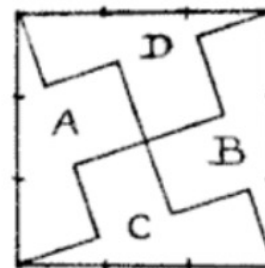
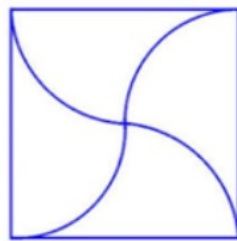
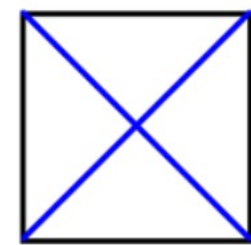
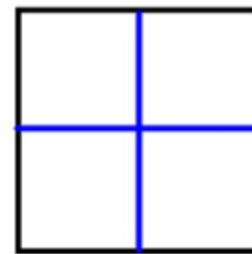
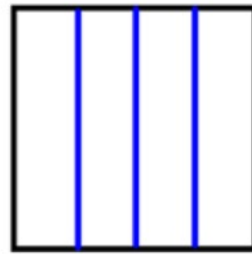


FIG. 18.

taken from:  
<https://www.bookofproofs.org/branches/greek-cross-puzzles/>

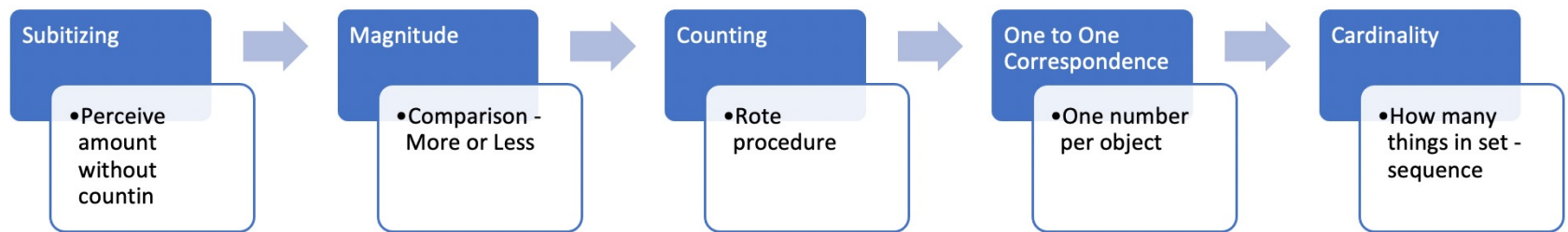


A decorative vertical bar on the left side of the slide, composed of two stacked rectangular sections. The top section is a medium blue color and contains a white Greek letter pi ( $\pi$ ). The bottom section is a darker blue color and is empty.

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What are numbers?

# Numeric Learning Trajectory



Douglas H. Clements, J. S. (2009). Learning and Teaching Early Math - The Learning Trajectories Approach. New York: Routledge.

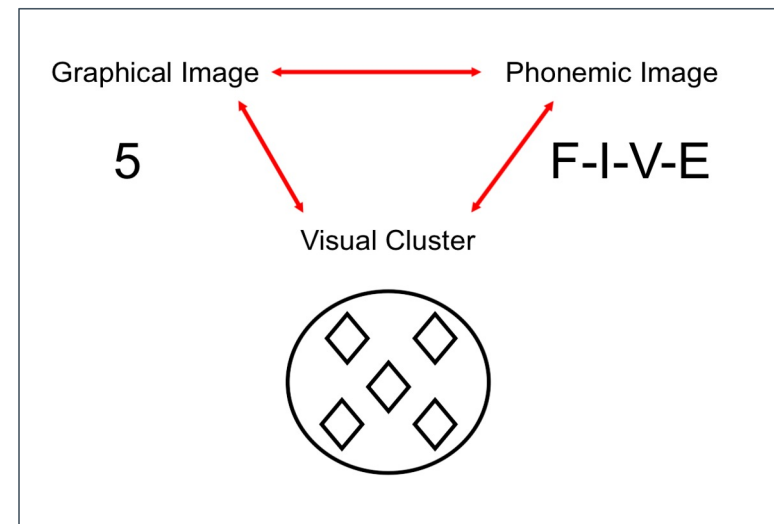


# Reading and Mathematics Connected

## Literacy and Numeracy – Developmental Parallels...

- “In reading one needs to focus on the phonemes (units of sound) in a word; in math, one needs to see cluster of objects in the mind’s eye.”

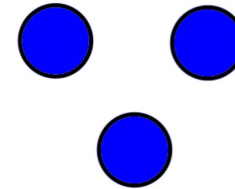
*The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties.* (2015). (S. Chinn Ed. First ed.). New York, NY: Routledge (p. 281)



# Symbols, Numerical Representation

1. Analogue Magnitude - Independent of Language

2. Written Words: Three



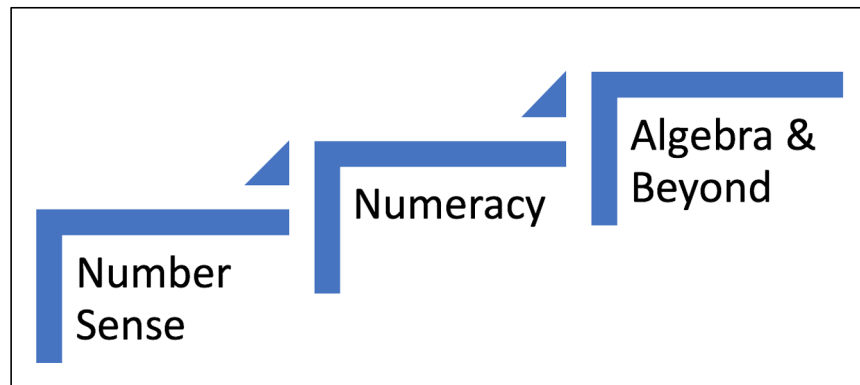
3. Arabic Numerals: 3

Fritz, A., Haase, V. G., & Räsänen, P. (2019). *International Handbook of Mathematical Learning Difficulties From the Laboratory to the Classroom: From the Laboratory to the Classroom*. (p. 9)

# Number Sense and Numeracy

“The ability to make sense of numbers and to use them effectively in real life situations. The basis of numeracy is number sense: understanding what numbers represent and how they can be used to solve problems.”

Jane Emerson, P. B. (2015). *Understanding Dyscalculia and Numeracy Difficulties: A Guide for Parents, Teachers and Other Professionals* (First ed.) (p 35).



## Basic Numeracy Skills

“Basic numerical thinking requires the following knowledge and skills.”

- **Counting** – accurately and flexibly
- Understanding **Number Relationships**
- Understanding **Place Value**
- **Calculation** – Problem Solving that involves processing #s

Jane Emerson, P. B. (2015). *Understanding Dyscalculia and Numeracy Difficulties: A Guide for Parents, Teachers and Other Professionals* (First ed.) (p 36).

## Basic Numeracy Skills

“Basic numerical thinking requires the following knowledge and skills.”

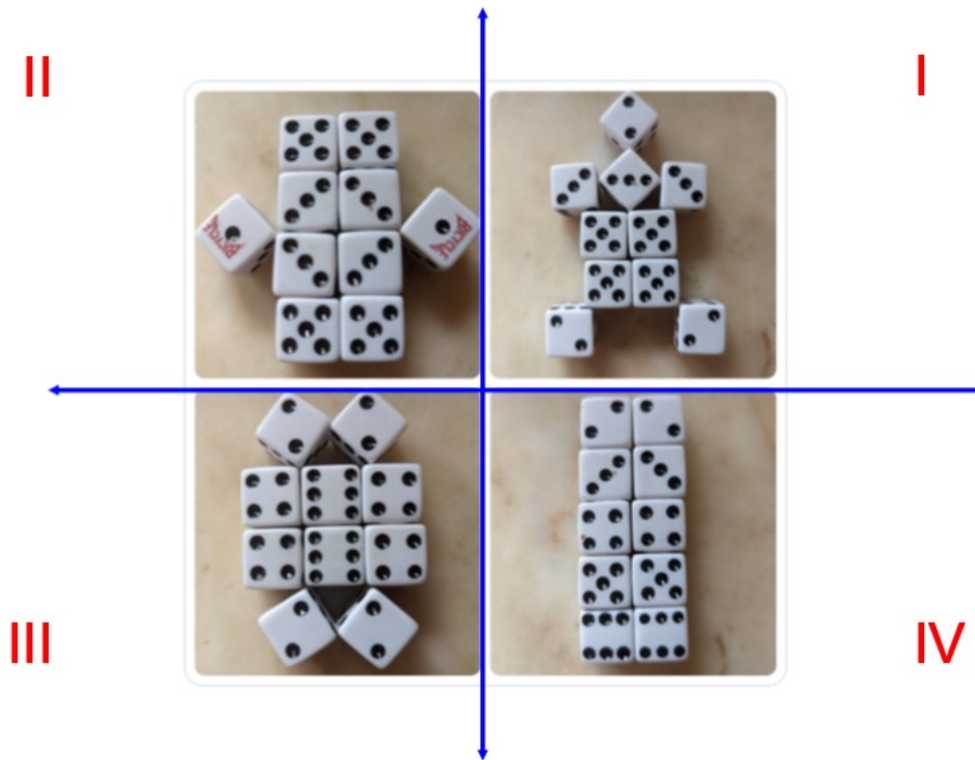
- Knowing **Key Facts**
- **Generalizing from Know Facts** – to Derive New Facts
- Understanding difference between **Counting numbers** and **Measuring numbers**
- **Numerical Thinking** – Logical Reasoning to Analyze Numerical Information

Jane Emerson, P. B. (2015). *Understanding Dyscalculia and Numeracy Difficulties: A Guide for Parents, Teachers and Other Professionals* (First ed.) (p 36).

## Task # 3 - Which One Doesn't Belong?

<https://wodb.ca/>

Quadrants:





## Low Floor High Ceiling Tasks...

- Allow accessibility for all students.
- Are engaging to experience.
- Extend to high levels that promotes deep thinking.
- Confirm that the problem is more important than the answer.
- Lead to rich (mathematical) discourse.
- Allow for differentiation based on interest, learning profile, readiness level.

Youcubed: <https://www.youcubed.org/tasks/>

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## Informal Example: Low Floor High Ceiling Tasks...

### What Makes a Good Task?

1. Non-Routine
2. Simply Stated (low floor)
3. Multiple Solutions (high ceiling)
4. Teacher Enjoys Solving it!



Visual Patterns - <https://www.visualpatterns.org/>

# Mathematical Myths:

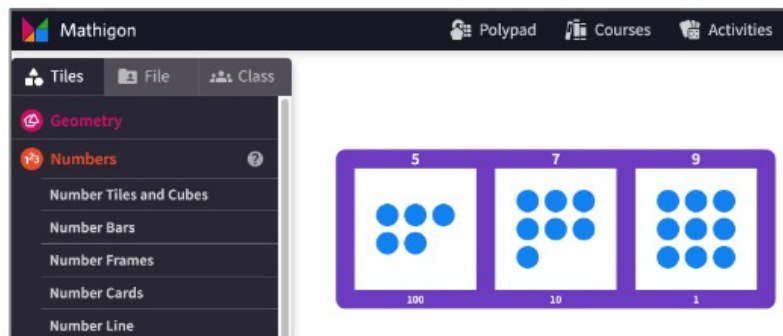
**Not True**

- Conceptual understanding should be mastered immediately.
- Procedural fluency should follow a scheduled time frame.

This is simply not how mathematical development works!

# The Purpose of Mathematics Education:

CONCEPTUAL UNDERSTANDING



PROCEDURAL FLUENCY

A handwritten long division problem on a piece of paper. The problem is  $4 \overline{) 579}$ . The quotient is written as 144, and the remainder is 3. Red arrows and labels identify the parts: 'Divisor' points to the 4, 'Dividend' points to the 579, 'Quotient' points to the 144, and 'Remainder' points to the 3. The steps of the division are shown: 4 goes into 5 one time (1), 4 goes into 17 four times (4), 4 goes into 19 four times (4), and the remainder is 3.

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I am going to ask you a serious question.

Ready?

Please be honest.

Have you, in the last month, not inside a school, but out in the public...

Been asked to do a long division problem by hand?

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## What do we know about Long Division? (Asking for a friend)!

Notice the question...what do we “know” – we all know something!

Rather than “What is Long Division” – seems like we would hear crickets with this question.



# What do we know about Long Division? (Asking for a friend)!

What do we know about long division?

Dr. Adair

Not Fun  
Confusing  
MULTISTEP

SUBTRACTION  
Time Consuming  
Long  
MULT  $\rightarrow$  SUBTRACT

	18	$\rightarrow$ Quotient
Divisor $\leftarrow$	4 $\overline{) 75}$	$\rightarrow$ Dividend
	- 4	
	<hr/>	
	35	
	- 32	
	<hr/>	
	03	$\rightarrow$ Remainder

## What is Long Division?

Long Division is an algorithm, series of steps, that allows us to break a large number into smaller groups of equal size.

What does 579 divided by 4 mean?

$$\begin{array}{r} 579 \\ \hline 4 \end{array}$$

A decorative vertical bar on the left side of the slide, consisting of a dark blue bar and a lighter blue bar. The Greek letter pi ( $\pi$ ) is centered in the lighter blue section.

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## Long Division – 7<sup>th</sup> Grade Lesson

1. We have stuff.
2. We want to put that stuff into groups.

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## Long Division – 7<sup>th</sup> Grade Lesson

1. We have stuff.
2. We want to put that stuff into groups.

What are the questions we need to ask?

How much stuff do we have?

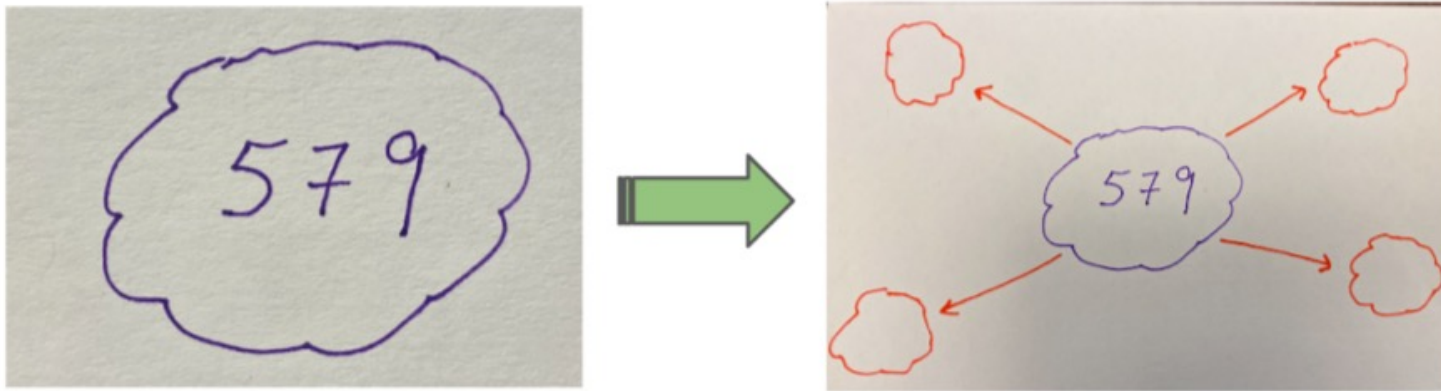
How many groups do we want?

Should the groups be equal in size?

# “Drill and Kill” VS Deep Mathematical Thinking

Conceptual Understanding:

We start with 579 units of something and we want 4 equal groups.



## We typically start with the procedural:

### Procedural Algorithm:

1. How many times does 4 go into 5?

Handwritten long division of 579 by 4. The divisor 4 is on the left, and the dividend 579 is on the right. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579. The quotient 1 is written above the 5. A horizontal line is drawn under the 5, and a 4 is written below it. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579.

2. Multiply 1 by 4 and subtract from 5; bring down the 7.

Handwritten long division of 579 by 4. The divisor 4 is on the left, and the dividend 579 is on the right. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579. The quotient 1 is written above the 5. A horizontal line is drawn under the 5, and a 4 is written below it. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579.

3. How many times does 4 go into 17?

Handwritten long division of 579 by 4. The divisor 4 is on the left, and the dividend 579 is on the right. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579. The quotient 1 is written above the 5. A horizontal line is drawn under the 5, and a 4 is written below it. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579.

4. Multiply 4 by 4 and subtract from 17; bring down the 9.

Handwritten long division of 579 by 4. The divisor 4 is on the left, and the dividend 579 is on the right. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579. The quotient 14 is written above the 57. A horizontal line is drawn under the 57, and a 16 is written below it. A red arrow points from the word "Divisor" to the 4, and another red arrow points from the word "Dividend" to the 579.

We typically start with the procedural:

5. How many times does 4 go into 19?

Handwritten long division of 19 by 4. The divisor 4 is on the left, and the dividend 19 is on the right. The quotient 4 is written above the 19. The first step shows 4 times 4 equals 16, which is subtracted from 19, leaving a remainder of 3. Red arrows point from the labels 'Divisor' and 'Dividend' to their respective parts in the problem.

So, what does this answer even mean?

6. Multiply 4 by 4 and subtract from 19. Three is left over. Does 4 go into 3?

Handwritten long division of 19 by 4. The divisor 4 is on the left, and the dividend 19 is on the right. The quotient 4 is written above the 19. The first step shows 4 times 4 equals 16, which is subtracted from 19, leaving a remainder of 3. Red arrows point from the labels 'Divisor', 'Dividend', and 'Remainder' to their respective parts in the problem.



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# “Drill and Kill” VS Deep Mathematical Thinking

Division Worksheet Packet				
Division Worksheet #1				
Divide the following.				
1. $9\overline{)45}$	2. $6\overline{)30}$	3. $4\overline{)32}$	4. $7\overline{)56}$	5. $9\overline{)63}$
6. $7\overline{)49}$	7. $3\overline{)24}$	8. $5\overline{)40}$	9. $6\overline{)42}$	10. $8\overline{)72}$
11. $5\overline{)20}$	12. $1\overline{)8}$	13. $7\overline{)56}$	14. $3\overline{)12}$	15. $9\overline{)54}$
16. $7\overline{)63}$	17. $8\overline{)0}$	18. $6\overline{)48}$	19. $2\overline{)18}$	20. $5\overline{)40}$
21. $6\overline{)18}$	22. $7\overline{)56}$	23. $8\overline{)56}$	24. $5\overline{)25}$	25. $9\overline{)81}$
26. $6\overline{)42}$	27. $4\overline{)32}$	28. $5\overline{)35}$	29. $4\overline{)28}$	30. $8\overline{)64}$

ONE TAKE AWAY – I BEG YOU!

Less Problems

More Space

PLEASE!

Conceptually:

We have stuff, we want groups.

$$579 \div 4$$

Diagram illustrating the components of the division problem  $579 \div 4$ :

- A red arrow points from the text "STUFF WE HAVE" to the dividend 579.
- A red arrow points from the text "GROUPS" to the divisor 4.

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Conceptually:

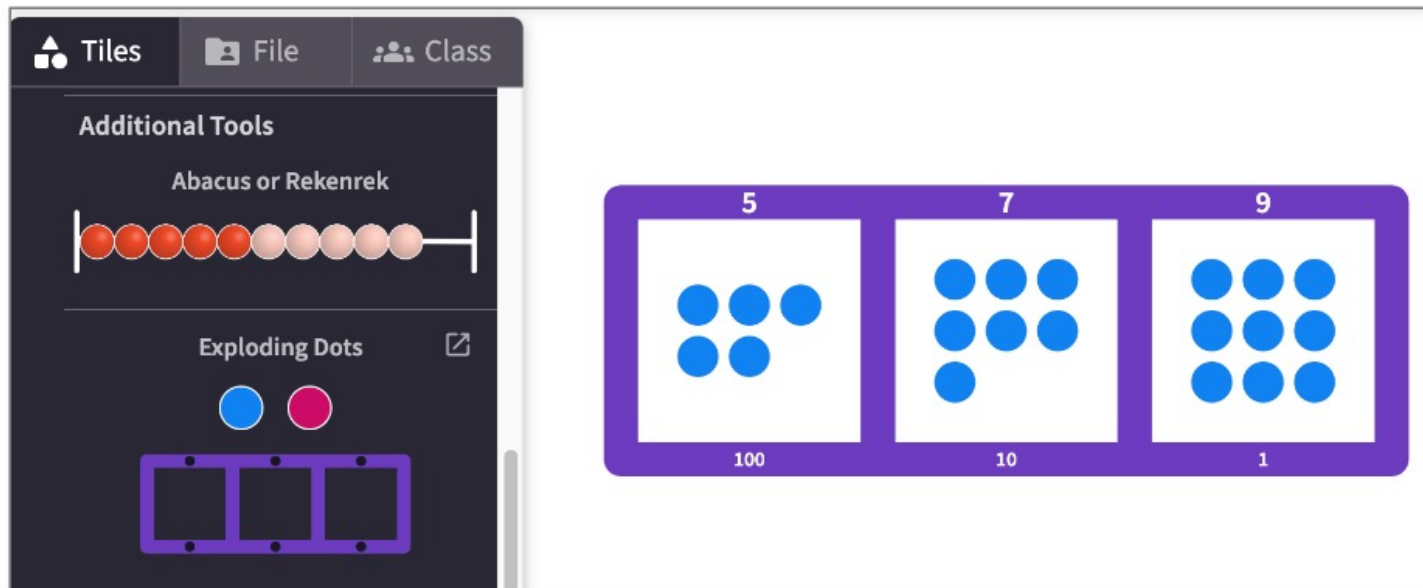
Mathigon can help students to work through this problem as a task.

Go to: [Mathigon.org](https://mathigon.org)

[Polypad](#) > [Tiles](#) > [Numbers](#) > [Additional Tools](#) > [Place Value Chart](#)

## Conceptually:

Grab the “Exploding” dots and drag into the “Place Value Chart” to make 579 units.



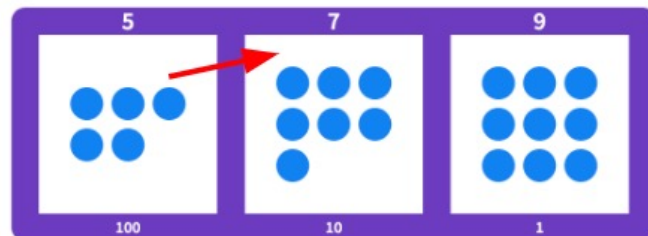
## Conceptually:

The “Exploding” Dots can be exchanged.

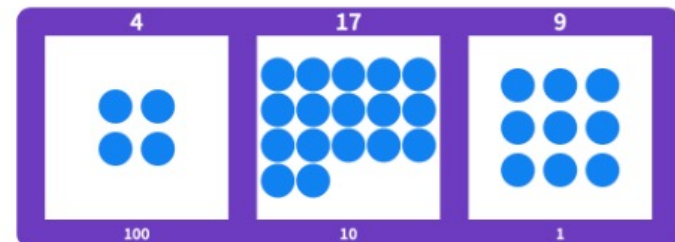
One dot in the 100s place can be exchanged for 10 dots in the tens place.

Notice the word choice!  
Not “Borrow”!

5 - 100s      7 - 10s      9 - 1s

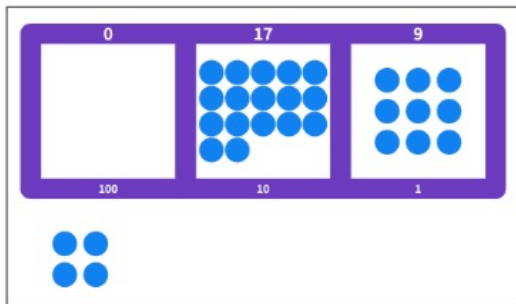


4 - 100s      17 - 10s      9 - 1s

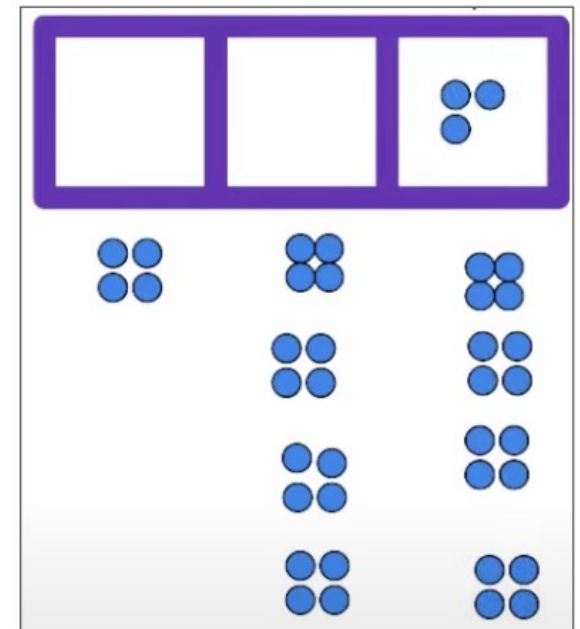
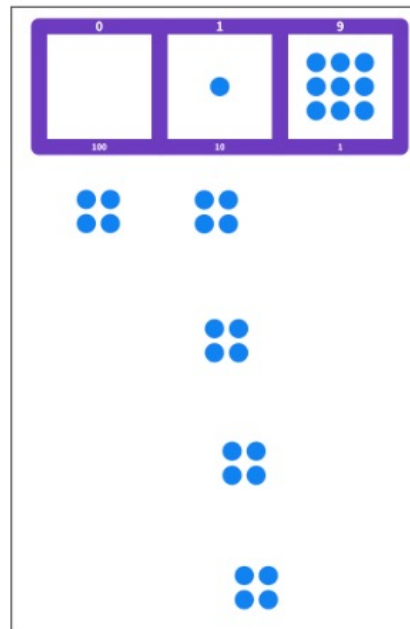


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Conceptually:



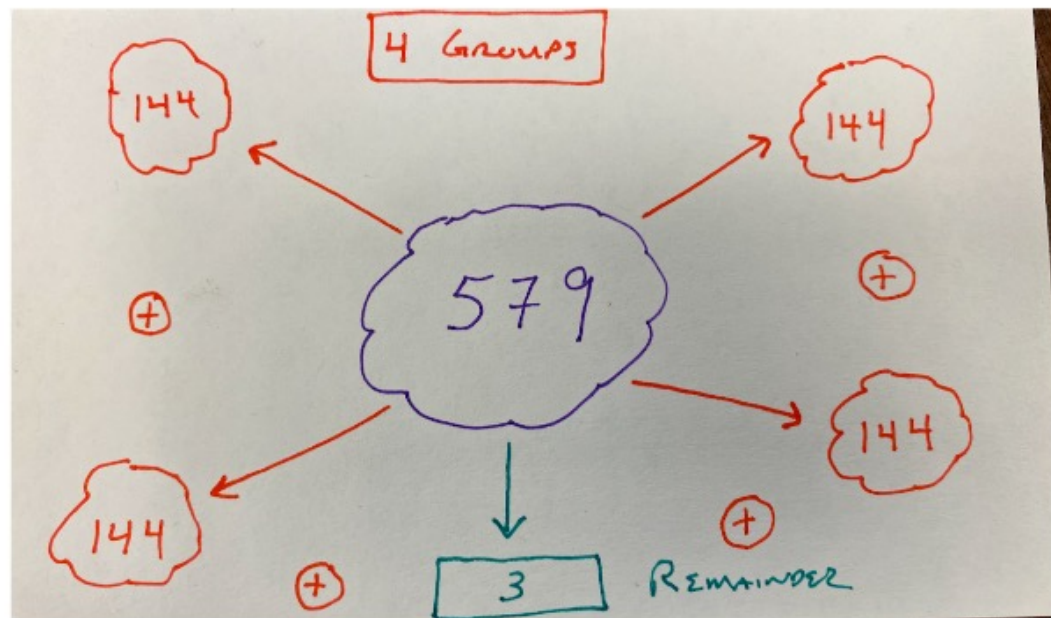
So, each group gets one hundred, four tens, four ones and we have three left over.



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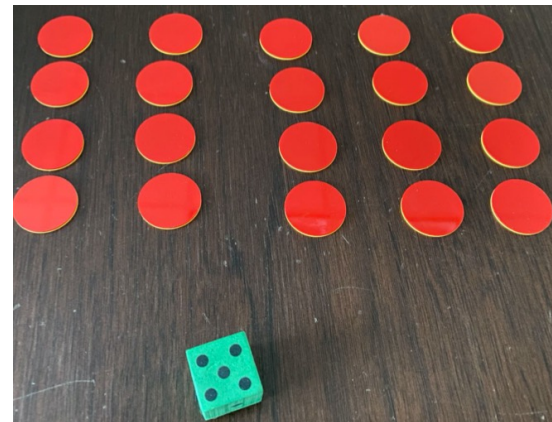
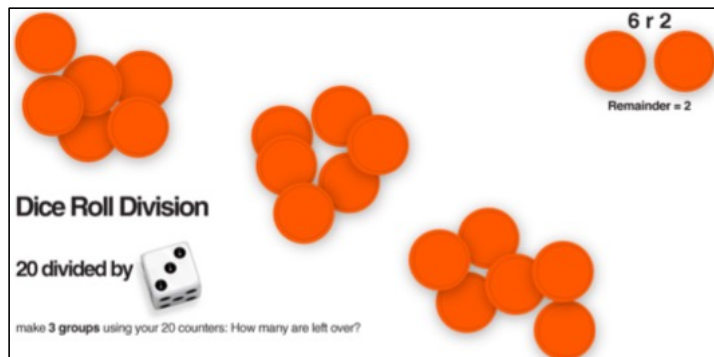
Conceptually:

$$579 \div 4$$



## Class Activity #1 – Dice Roll Division

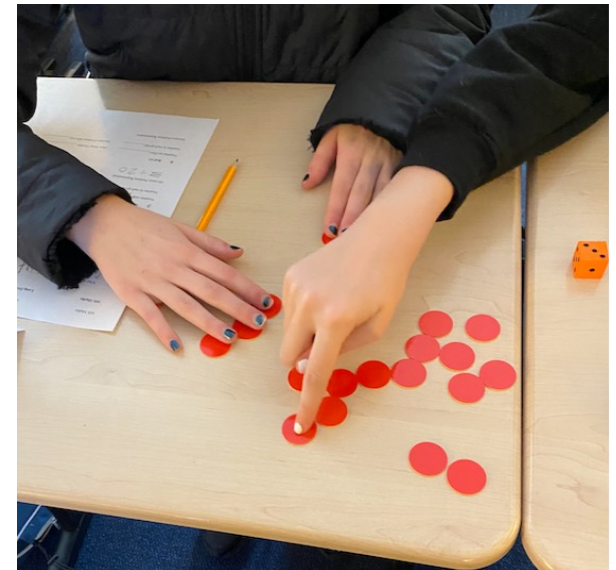
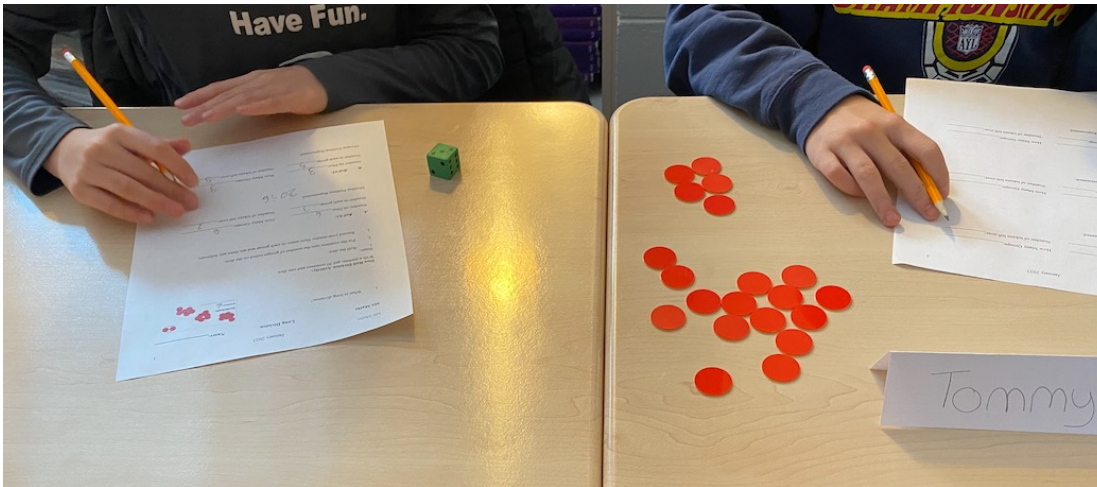
1. Get 20 tokens.
2. Get one Dice (number cube).
3. Roll the Dice - that is the number of groups you want.





$\pi$

## Class Activity – Dice Roll Division



Divide 20 into how many groups?

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So...  
the purpose  
is to...what  
again?



**Dan Anderson** @dandersod · 33m

...

@mpershan Lol, as I was reading this email, my 5th grader was sitting at the table doing his homework using long division for two-digit divisor into four-digit dividend and checking his work with a calculator (app on a phone).

He has all sorts of ideas for using calculators in the classroom, but he does have one word of warning:

I suggest one caution when using the calculator. The temptation is to have students use the calculator to check complex paper-and-pencil computation. Put yourself in the position of the student who has just spent five minutes **dividing a three-digit divisor into a five-digit dividend** and then checks the answer in a few seconds on the calculator. This student is convinced that paper-and-pencil computation is foolish and that the school

# We hope our students become “Mathematically Proficient”

## Five Strands:

1. Conceptual Understanding – comprehension of mathematical concepts, operations, and relations
2. Procedural Fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. Strategic Competence – ability to formulate, represent, and solve mathematical problems
4. Adaptive Reasoning – capacity for logical thought, reflection, explanation, and justification
5. Productive Disposition – habitual inclination to see maths as sensible, useful, and worthwhile with a belief in oneself

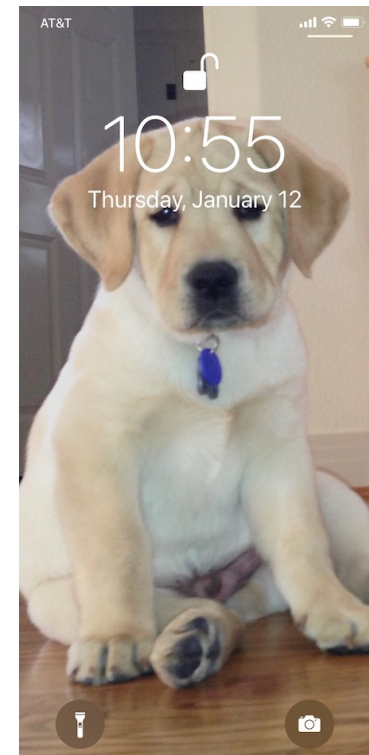
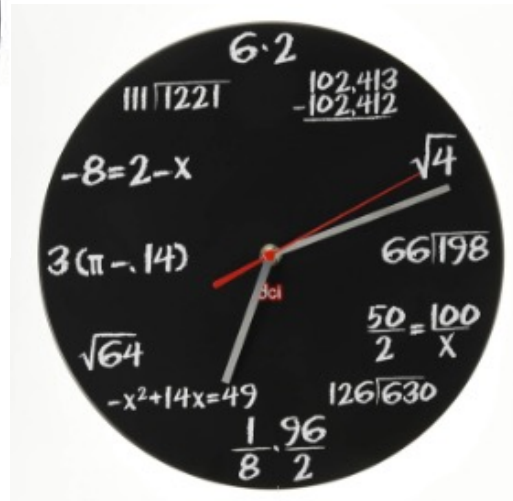
## Challenges that our students face...

- “Executive Functioning – the mental abilities we use to manage and regulate our behaviors; EF is essential for planning, organizing, strategizing, paying attention to and remembering details, and managing time and space.”
- “Working Memory – governs our ability to retrain and manipulate distinct pieces of information over short periods of time.”
- “Mental Flexibility – helps us to sustain or shift attention to different demands or to apply different rules in different settings.”
- “Self-Control – enables us to set priorities and resist impulsive actions or responses.” – Center on the Developing Child 2004

Jane Emerson, P. B. (2015). *Understanding Dyscalculia and Numeracy Difficulties: A Guide for Parents, Teachers and Other Professionals* (First ed.) (p 30).

$\pi$

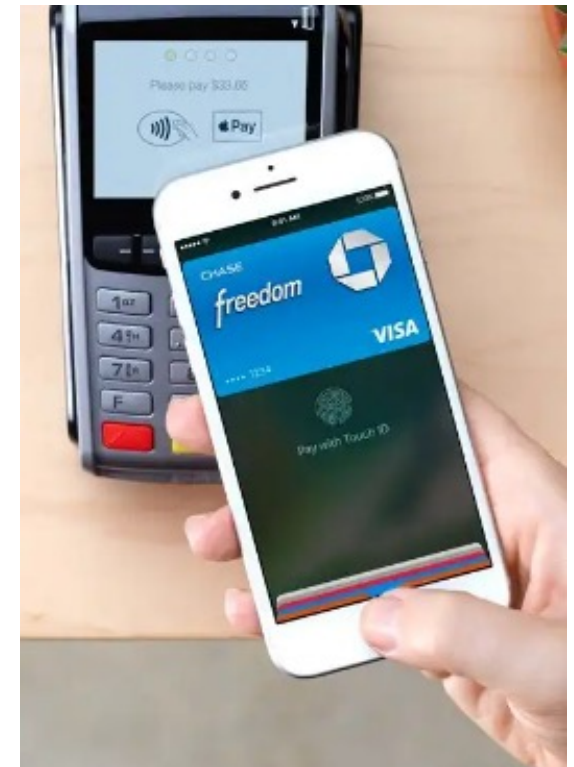
What we used to use...no longer works!



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$\pi$

What we used to use...no longer works!



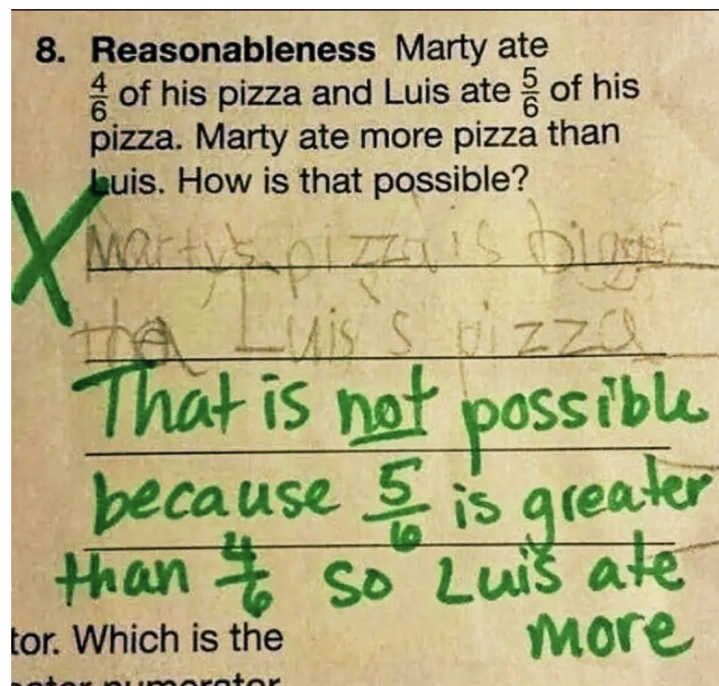
MINDY ADAIR, PH.D.

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Are we open to student thinking?

Or do we have one answer in mind?



Are we open to student thinking?

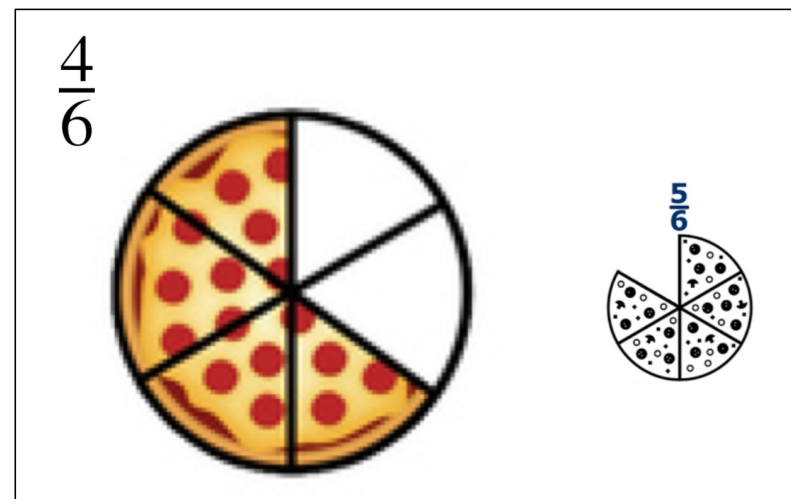
Or do we have one answer in mind?

8. **Reasonableness** Marty ate  $\frac{4}{6}$  of his pizza and Luis ate  $\frac{5}{6}$  of his pizza. Marty ate more pizza than Luis. How is that possible?

*Marty's pizza is bigger than Luis's pizza*

*That is not possible because  $\frac{5}{6}$  is greater than  $\frac{4}{6}$  so Luis ate more*

tor. Which is the  
ator numerator





$\pi$

## My nephew...and a story...



## My nephew...and a story...



School Year 2020 - 2021  
9<sup>th</sup> Grade

**Failing Algebra I – as in an F.**

Tons of tutoring, hours of support.

Earned an A- (90%) on a test.

Earned an 83% on the final exam!

School Year 2021 – 2022:  
Geometry. Problems for sure!

$\pi$

That was then...




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$\pi$ 

This is now...



Weight: 25	77.29%	+
<b>Test</b>	388.25/475	-
Weight: 40	(81.73%)	
 TEST Ch. 6	92/100	>
Due: 11/30/2022	(92%)	
Got a 92 on the chapter 6 test		

FANTASTIC!!!!!! Thanks for sharing - I was in a meeting otherwise I would have responded sooner.YAHOOOOOOOOOOOOO - so proud of you Mas. Congratulations.

Got an 84 on the math final

## Task #4:

What do you notice? What do you wonder?

SNOWIEST TOWNS			COLORADO 2022-23			DEC. 31
	2022-23	LAST SEASON DIFF		2022-23	LAST SEASON	
WINTER PARK	92.3"	+8.8"	CRESTED BUTTE	54.2"	-58.6"	
STEAMBOAT SPRINGS	92.1"	+46.0"	OURAY	50.9"	+28.3"	
MESA	79.5"	+29.5"	ASPEN	45.2"	-24.5"	
GRAND LAKE	73.9"	+20.3"	FRASER	43.9"	+37.9"	
HAYDEN	71.3"	+25.3"	COLLBRAN	42.4"	+27.2"	
VAIL	59.8"	+8.5"	BOULDER	40.2"	+34.4"	

## Class Activity #2 – Tallest Tower

10 Index Cards and one unopened post-it-notes

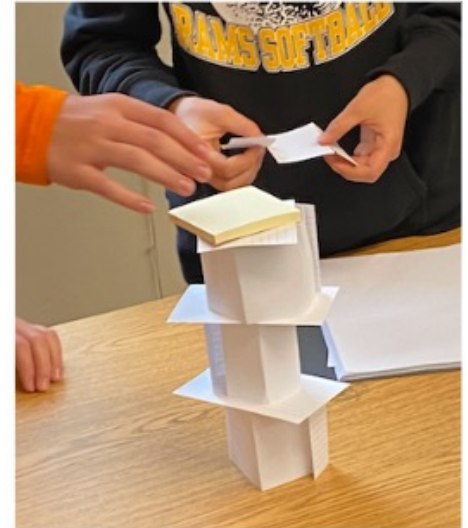
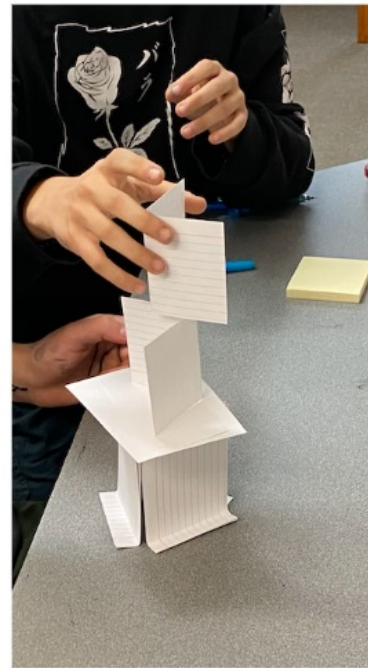
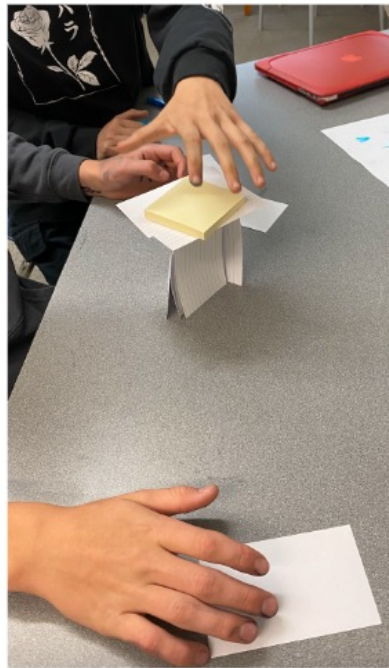
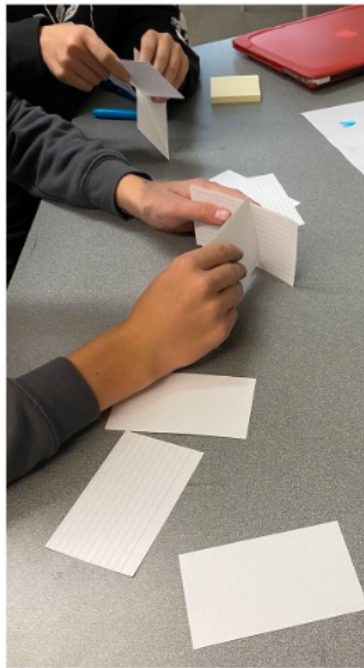


The Goal: Make the tallest free-standing tower.



$\pi$

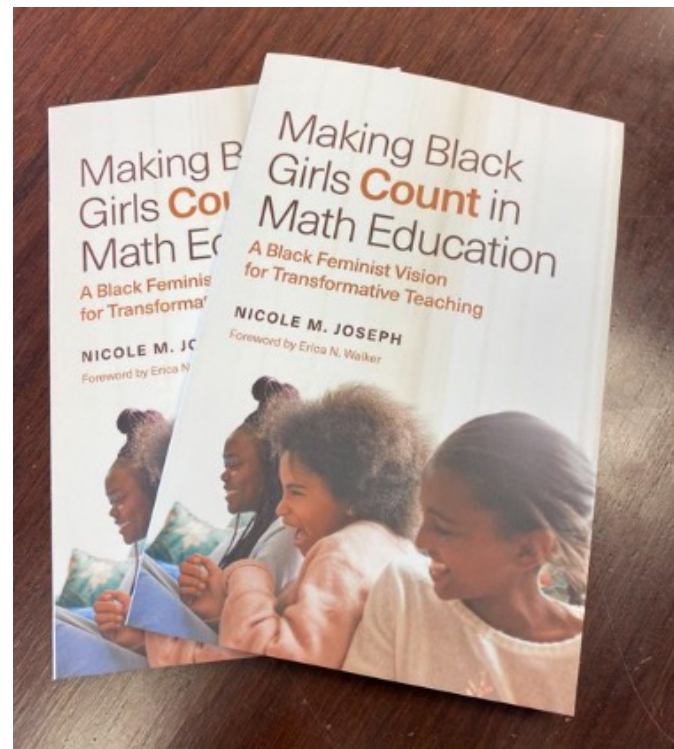
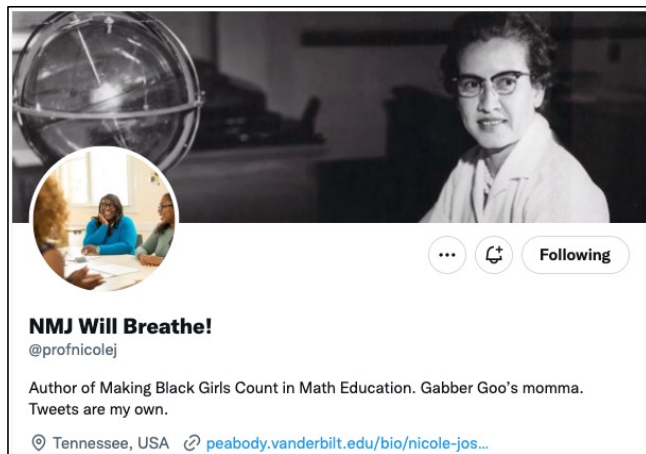
## Class Activity – Tallest Tower



$\pi$

# Who *counts* in maths education?

@profnicolej



MINDY ADAIR, PH.D.



# Who *counts* in the world around us?

## Kuno the Servicerottie 🇨🇦 🐕 🦽

@servicerotties

I'm Kuno, a Gov't qualified service dog in Alberta, Canada. My job is taking care of my person, a bilateral amputee, [#weatherdog](#) & Spokesdog for the disabled

📁 Social Media Influencer 📍 Alberta, Canada 🔗 [servicerotties.com](https://servicerotties.com)  
📅 Joined April 2020

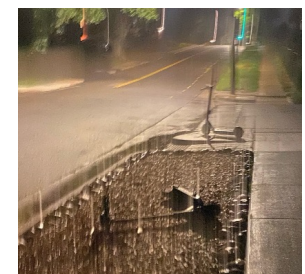
9,823 Following 27.5K Followers



Kuno the Servicerottie 🇨🇦 🐕 🦽 @servicerotties · Jul 20

If you come across these rental bikes or scooters left on the sidewalk, could you please move them for those that can't? We could get around it to the curb ramp with a wheelchair, but there wasn't room to get by if we wanted to turn the corner instead.

Thanks for helping. 🙏



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## Task #5: A Tape Measure and a Bolt... Disappointing – or – Cool?



## Task #5 – A Tape Measure and a Bolt... Cool!!



$\pi$

























How Many? How would you count?





























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What do you Notice? Wonder?

 <b>#4-40*1/4"</b> 25pcs	 <b>#4-40*5/16"</b> 25pcs	 <b>#4-40*3/8"</b> 25pcs	 <b>#4-40*5/8"</b> 25pcs	 <b>#4-40</b> 50pcs	 <b>#4-40</b> 50pcs
 <b>#6-32*1/4"</b> 25pcs	 <b>#6-32*3/8"</b> 25pcs	 <b>#6-32*5/8"</b> 25pcs	 <b>#6-32*3/4"</b> 25pcs	 <b>#6-32</b> 50pcs	 <b>#6-32</b> 50pcs
 <b>#8-32*1/4"</b> 20pcs	 <b>#8-32*1/2"</b> 20pcs	 <b>#8-32*5/8"</b> 20pcs	 <b>#8-32*1"</b> 15pcs	 <b>#8-32</b> 45pcs	 <b>#8-32</b> 45pcs
 <b>#10-24*3/8"</b> 15pcs	 <b>#10-24*5/8"</b> 10pcs	 <b>#10-24*3/4"</b> 10pcs	 <b>#10-24*1"</b> 10pcs	 <b>#10-24</b> 45pcs	 <b>#10-24</b> 45pcs

$$25+25+25+25= 100$$

 #4-40*1/4" 25pcs	 #4-40*5/16" 25pcs	 #4-40*3/8" 25pcs	 #4-40*5/8" 25pcs	 #4-40 50pcs	 #4-40 50pcs
 #6-32*1/4" 25pcs	 #6-32*3/8" 25pcs	 #6-32*5/8" 25pcs	 #6-32*3/4" 25pcs	 #6-32 50pcs	 #6-32 50pcs
 #8-32*1/4" 20pcs	 #8-32*1/2" 20pcs	 #8-32*5/8" 20pcs	 #8-32*1" 15pcs	 #8-32 45pcs	 #8-32 45pcs
 #10-24*3/8" 15pcs	 #10-24*5/8" 10pcs	 #10-24*3/4" 10pcs	 #10-24*1" 10pcs	 #10-24 45pcs	 #10-24 45pcs

$$15+10+10+10 = 45$$

$\pi$

#4, #6, #8, #10 is an Arithmetic Sequence

#4-40*1/4" 25pcs	#4-40*5/16" 25pcs	#4-40*3/8" 25pcs	#
#6-32*1/4" 25pcs	#6-32*3/8" 25pcs	#6-32*5/8" 25pcs	#
#8-32*1/4" 20pcs	#8-32*1/2" 20pcs	#8-32*5/8" 20pcs	
#10-24*3/8" 15pcs	#10-24*5/8" 10pcs	#10-24*3/4" 10pcs	#

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{3}{8}$$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{5}{8}$$

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\frac{3}{4} = \frac{6}{8}$$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

Fractions – Comparison  
and Finding LCD,  
Equivalent Fractions with  
[Mathigon.org](http://Mathigon.org)

# We hope our students become “Mathematically Proficient”

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1. Conceptual Understanding – comprehension of mathematical concepts, operations, and relations
2. Procedural Fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. Strategic Competence – ability to formulate, represent, and solve mathematical problems
4. Adaptive Reasoning – capacity for logical thought, reflection, explanation, and justification
5. Productive Disposition – habitual inclination to see maths as sensible, useful, and worthwhile with a belief in oneself



## Class Activity #3 – Open Middle

### DIFFERENCE OF SQUARES AND SUM OF CUBES

Directions: Using the digits 0 to 9 at most one time each, place a digit in each box to make both expressions factorable.

$$\square\square x^2 - \square\square y^2$$

$$\square\square a^3 + \square\square b^3$$

<https://www.openmiddle.com/>

## Class Activity #3 – Open Middle

Recall:

Difference of Squares

Sums of Cubes

$$x^2 - y^2 = (x + y)(x - y)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

<https://www.openmiddle.com/>

## Class Activity #3 – Open Middle

$$\boxed{4}\boxed{9}x^2 - \boxed{1}\boxed{6}y^2$$

$$\boxed{0}\boxed{8}a^3 + \boxed{2}\boxed{7}b^3$$

Check:

$$49x^2 - 16y^2 = (7x + 4y)(7x - 4y)$$

$$08a^3 + 27b^3 = (2a + 3b)(4a^2 - 6ab + 9b^2)$$

$T^3 = \text{Tasks, Tech, Twitter}$

## TASKS

Low Flow High Ceiling (LFHC)

- Fawn Nguyen's Visual Patterns - <https://www.visualpatterns.org/>
- Between Two Numbers - <https://www.between2numbers.com/>
- Chewable Math - <https://www.chewablemath.com/>
- Mary Bourassa's Which One Doesn't Belong - <https://wodb.ca/>

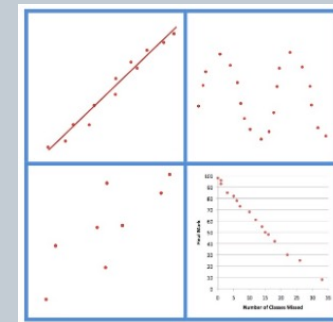
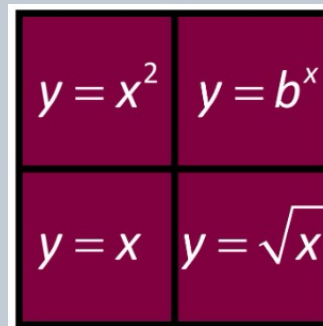
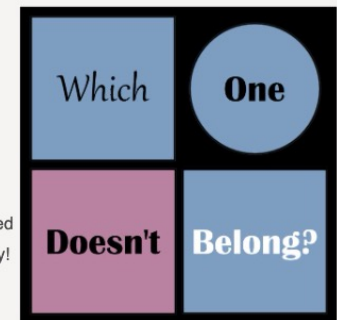


## WHICH ONE DOESN'T BELONG?

### THIS WEBSITE WAS INSPIRED BY THE MTBOS

with special thanks to Christopher Danielson and his Which One Doesn't Belong - A Shapes Book.

This is **Which One Doesn't Belong?**, a website dedicated to providing thought-provoking puzzles for math teachers and students alike. There are no answers provided as there are many different, correct ways of choosing which one doesn't belong. Enjoy!



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$\pi$

# Dailey Sets - <https://www.setgame.com/set/puzzle>

SETS Found: ☐

Hover to Enlarge


Download our interactive flash tutorial

$T^3 = \text{Tasks, Tech, Twitter}$

## TECHNOLOGY

Tech Resources:

- Mathematical Playground – Mathigon - <https://mathigon.org/>
- Desmos – Free Graphing Calculator and Activities - <https://www.desmos.com/>
- You Cubed - <https://www.youcubed.org/>

### Polypad

Unleash your creativity with the world's best manipulatives! Engage in problem-solving, explore patterns and collaborate with others.

Launch Polypad →

Lesson Plans →

Tutorials →




$\frac{1}{2}$	$\frac{1}{2}$			
0.5	0.5			
50%	50%			
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
0.2	0.2	0.2	0.2	0.2
20%	20%	20%	20%	20%

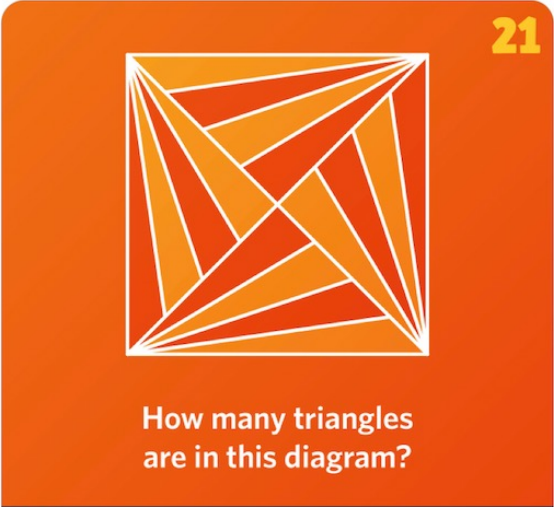
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# Activities and Puzzles – can expand thinking!

 **Mathigon** @MathigonOrg · 3h ...  
DAY 21: How many triangles are in this diagram?

All puzzles and solutions at  
[mathigon.org/puzzles](https://mathigon.org/puzzles)

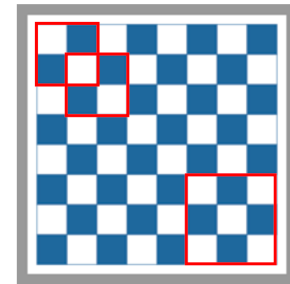
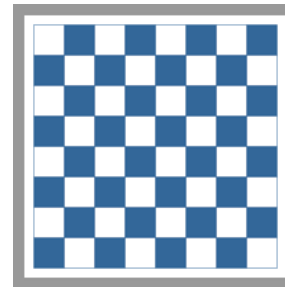


How many triangles are in this diagram?

 Mathigon

Thinking about our 2E students...how we can challenge them...

How many squares on a checkerboard?

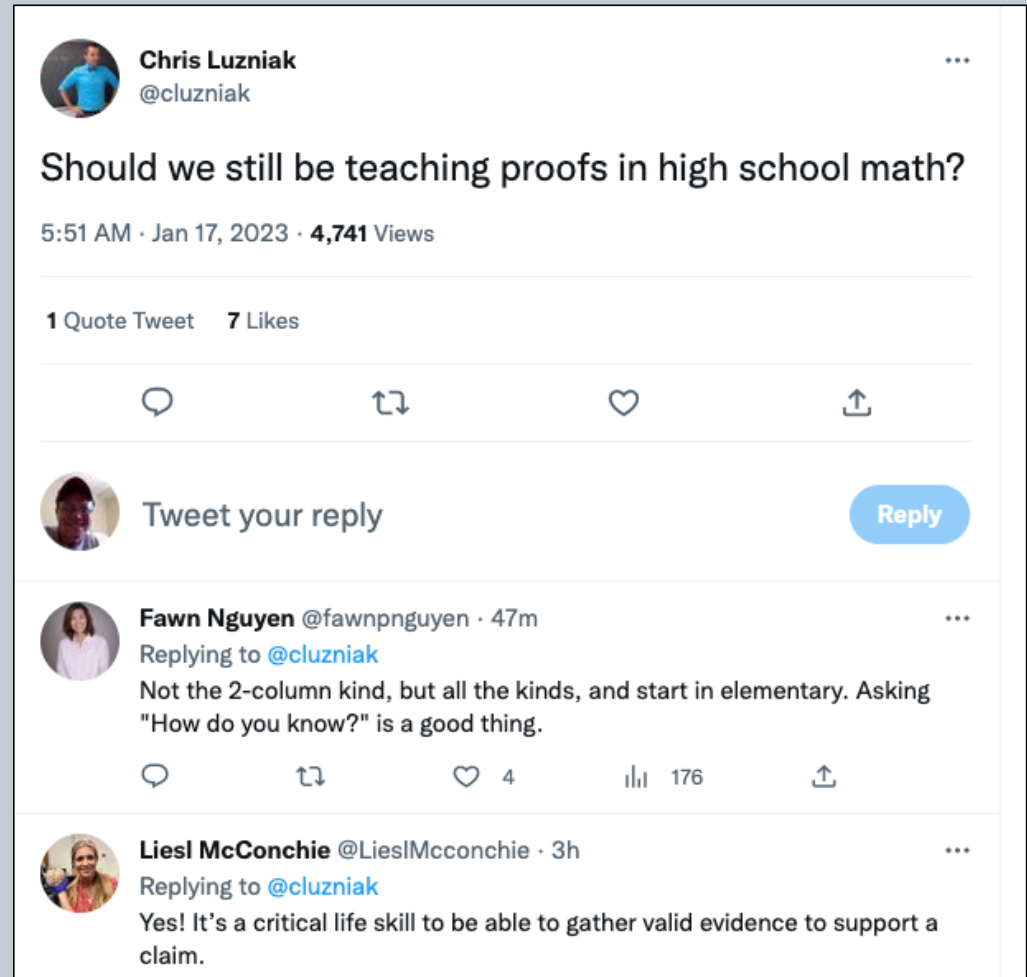


$T^3 = \text{Tasks, Tech, Twitter}$

## TWITTER

### Mathematics Educators

- Fawn Nguyen [@fawnpnguyen](#)
- Howie Hua [@howie\\_hua](#)
- Robert Kaplinsky  
[@robertkaplinsky](#)
- Dan Meyer [@ddmeyer](#)
- Sunil Singh [@Mathgarden](#)
- Christopher Danielson  
[@Trianglemancsd](#)
- Dr. Nicole Joseph [@profnicolej](#)
- Douglas H. Clements  
[@DHClements](#)

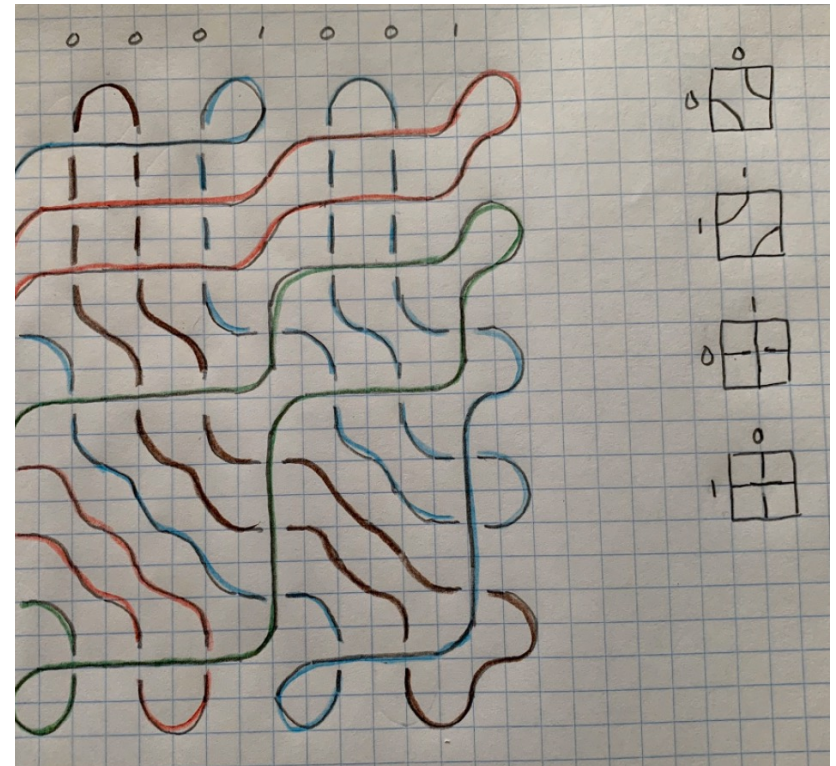
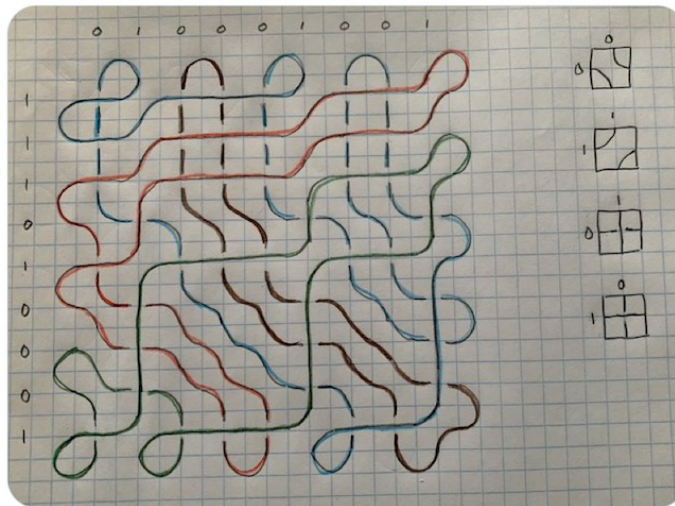






**Dave Richeson** @divbyz... · 4/5/20 ...

While walking the dog I had the idea for this modification of @anniek\_p's [#mathartchallenge](#) activity. Randomly generate zeros and ones and use them to determine which of the four knot tiles to put in the corresponding cell. Alternate joining the end along the outside. I 1/2



$\pi$

Warm Up:


The answer is...2023.

C. How can you use the set of numbers from 1 to 10 and mathematical operations to get 2023?

$$(10 + (9 + 8 \cdot 7) \cdot 6) \cdot 5 + 4 \cdot 3 \cdot 2 - 1 = 2023$$

# NCTM – Play the 2023 Year Game!

<https://www.nctm.org/Classroom-Resources/Year-Game/The-Year-Game/>

 **NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS**

[Classroom Resources](#) | [Publications](#) | [Standards & Positions](#) | [Advocacy](#) | [Research](#) | [Conferences](#)

## Play the 2023 Year Game

**The Year Game**  
[Rules of the Year Game](#)  
[Activity Sheet](#)  
[Manipulatives](#)

### Play the 2023 Edition Now!

Check out the 2023

- [rules](#)
- [activity sheet](#)
- [manipulatives](#)

## Rules

Use the digits in the year 2023 and the operations  $+$ ,  $-$ ,  $\times$ ,  $\div$ , sqrt (square root),  $^$  (raise to a power),  $!$  (factorial), and  $!!$  (double factorial) along with grouping symbols, to write expressions for the counting numbers 1 through 100.

1. All four digits must be used in the expression.
2. Only the digits 2, 0, 2, 3 may be used.

## The Year Game Activity Sheet 2023

1	35	69
2	36	70
3	37	71
4	38	72
5	39	73
6	40	74
7	41	75
8	42	76
9	43	77
10	44	78
11	45	79
12	46	80
13	47	81
14	48	82
15	49	83
16	50	84
17	51	85
18	52	86
19	53	87
20	54	88
21	55	89
22	56	90

MINDY ADAIR, PH.D.

$\pi$

## PD: Math on the Planes!

Feb. 24<sup>th</sup> & 25<sup>th</sup>

Presenting

Dr. Mindy Adair



**Title:** Making Mathematics Meaningful By Expanding Your Identity as a Mathematician

Presenting Sponsor



**Location:**

Denver Academy

4400 E. Iliff Avenue, Denver, CO 80222



MINDY ADAIR, PH.D.

# WHAT ABOUT MATHS?




Sponsored by the  
Colorado Department of  
Education!

April 11, 2023

Elementary **Free** Webinar  
(45 minutes)

April 12, 2023

Secondary **Free** Webinar  
(45 minutes)

	
<b>COLORADO</b> Department of Education	
<b>What About Maths?</b> Webinar Series Let's Talk Mathematics!	
<u>WHAT:</u> Two 45 minute live webinars will address how to facilitate student engagement through mathematical discourse for deeper learning.	
<u>WHEN:</u> Elementary focus: Wednesday 04/11/23 from 4:00 to 4:45 pm Secondary focus: Thursday 4/12/2023 from 4:00 to 4:45 pm	
<u>WHERE:</u> A Zoom link to attend the webinar will be sent to you.	
<b>Webinar Objectives:</b> <ul style="list-style-type: none"><li>• Develop teacher questioning skills that will support, rather than sabotage, student learning.</li><li>• Learn how research-based practices encourage student discourse.</li><li>• Examine strategies for facilitating classroom discussion that will increase conceptual understanding.</li></ul>	
<b>Registration links below:</b>	
<b>Elementary Focus</b> 	<b>Secondary Focus:</b> 



# Session Handout

Mindy Adair, Ph.D.

1

**Courage to Risk – January 20, 2023**



**Mindy Adair, Ph.D.**

madair@denveracademy.org

**Director of Mathematics; Technology Integration Specialist - Denver Academy**

***Understanding Students' Mathematical Development: Let's Do This!***

All of our students deserve the opportunity to explore, grapple with, and learn mathematics. The way it was, does not have to be the way it is. Let's take this opportunity to investigate how mathematics can be accessible to all students. Maths is for everyone!

$T^3 = \text{Tasks, Tech, Twitter!}$

**A. High Quality Mathematical Tasks Are:**

- Designed to give students the space and opportunity to think and learn
- Differentiable and engaging
- Aligned with content and promote curiosity

**B. Technology should be:** (Fred Davis 1989 – Technology Acceptance Model – TAM)

- Easy to Use
- Useful

**C. Twitter: Mathematics Educators: Researchers and Practitioners**

- Research can inform our thinking.
- Experience in the classroom can expand our ideas.
- Learning about our students can guide our practice.

**A. Tasks to consider. Low Floor High Ceiling Tasks**

- Fawn Nguyen's Visual Patterns - <https://www.visualpatterns.org/>
- Between Two Numbers - <https://www.between2numbers.com/>
- Chewable Math - <https://www.chewablemath.com/>
- Mary Bourassa's Which One Doesn't Belong - <https://wodb.ca/>
- Robert Kaplinsky's Open Middle - <https://www.openmiddle.com/>
- Daily Set Puzzle - <https://www.setgame.com/set/puzzle>
- Andrew Stadel's Estimation 180 - <http://www.estimation180.com/>
- Balance Puzzles - <https://solve.me.edc.org/>
- Nat Banting's Fraction Talks - <http://fractiontalks.com/>

**B. Helpful Technological Resources:**

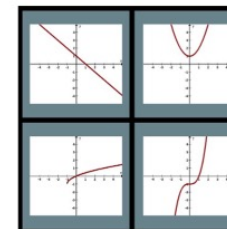
- Mathematical Playground – Mathigon - <https://mathigon.org/>
- Desmos – Free Graphing Calculator and Activities - <https://www.desmos.com/>
- You Cubed - <https://www.youcubed.org/>
- NCTM Classroom Resources - <https://www.nctm.org/classroomresources/>
- NCTM Illuminations - <https://illuminations.nctm.org/>
- University of Colorado – PhET - <https://phet.colorado.edu/>
- GeoGebra - <https://www.geogebra.org/>

**C. Mathematics Educators on the Twitter:**

- Fawn Nguyen @fawnpnguyen
- Howie Hua @howie\_hua
- Robert Kaplinsky @robertkaplinsky
- Dan Meyer @ddmeyer
- Sunil Singh @Mathgarden
- Christopher Danielson @Triangleman
- Mary Bourassa @MaryBourassa
- Dr. Nicole Joseph @profnicolej
- Douglas H. Clements @DHClements
- Amanda Davey @ADyscalculia
- Mathigon @MathigonOrg
- S. Leigh Nataro @mathteacher24
- Sarah Powell @sarahpowellphd
- Dr. Cathery Yeh @YehCathery
- Steve Chinn @stevechinnUK
- Judy Hornigold @DyscalculiaInfo
- The Dyscalculia Network @DyscalculiaNet1

**Bonus Follows:**

- Kuno the Servicerottie @servicerotties
- Marla Smith (Kuno's human) @Marla\_jsmith
- WeRateDogs @dog\_rates
- CanadianPenny @CanadianPenny1



# Closing

Fill in the blank:

Mathematical \_\_\_\_\_





## Closing

- Mathematical [Knowledge](#)
- Mathematical [Content](#)
- Mathematical [Skills](#)
- Mathematical [Goals](#)
- Mathematical [Speed](#)
- Mathematical [Identity](#)
- Mathematical [Prowess](#)
- Mathematical [Confidence](#)
- Mathematical [Trajectory](#)
- Mathematical [Performance](#)
- Mathematical [Anxiety](#)
- Mathematical [Confusion](#)
- Mathematical [Distain](#)
- Mathematical [Grades](#)
- Mathematical [Disability](#)
- Mathematical [Thinking](#)

Presentation Handout  
and Slides - PDFs at:

<https://www.mathsforallradair.org/>

Thank you! Questions?

Mindy Adair, Ph.D.

Director of Mathematics - Denver Academy

madair@denveracademy.org    @AdairMindy

$$P_{ni} \left( x_{ni} = \frac{1}{B_n}, D_i \right) = \frac{e^{(B_n - D_i)}}{1 - e^{(B_n - D_i)}}$$

